

The symbols P , Q , and R denote arbitrary statements, and **T** and **F** denote “True” and “False” respectively. The following are fundamental logical equivalences.

Name	Equivalence
Exclusive Middle	$P \vee (\neg P) \equiv \mathbf{T}$ $P \wedge (\neg P) \equiv \mathbf{F}$
Double Negation Law	$\neg(\neg P) \equiv P$
Idempotence Laws	$P \vee P \equiv P$ $P \wedge P \equiv P$
Identity Laws	$P \wedge \mathbf{T} \equiv P$ $P \vee \mathbf{F} \equiv P$
Domination Laws	$P \vee \mathbf{T} \equiv \mathbf{T}$ $P \wedge \mathbf{F} \equiv \mathbf{F}$
Commutative Laws	$P \vee Q \equiv Q \vee P$ $P \wedge Q \equiv Q \wedge P$
Associative Laws	$P \vee (Q \vee R) \equiv (P \vee Q) \vee R$ $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$
Distributive Laws	$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$ $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
De Morgan's Laws	$\neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q)$ $\neg(P \wedge Q) \equiv (\neg P) \vee (\neg Q)$
Material Implication	$P \rightarrow Q \equiv (\neg P) \vee Q$
Contraposition	$P \rightarrow Q \equiv (\neg Q) \rightarrow (\neg P)$
Biconditional Expansion	$P \leftrightarrow Q \equiv (P \rightarrow Q) \wedge (Q \rightarrow P)$

The following are fundamental logical deduction rules.

Name	Inference Rule
Modus Ponens	$P \rightarrow Q, P \therefore Q$
Modus Tollens	$P \rightarrow Q, \neg Q \therefore \neg P$
Disjunctive Syllogism	$P \vee Q, \neg P \therefore Q$
Hypothetical Syllogism	$P \rightarrow Q, Q \rightarrow R \therefore P \rightarrow R$
Dilemma	$P \vee Q, P \rightarrow R, Q \rightarrow R \therefore R$
Reductio Ad Absurdum	$(\neg P) \rightarrow [Q \wedge (\neg Q)] \therefore P$
Conjunctive Simplification	$P \wedge Q \therefore P$
Conjunctive Addition	$P, Q \therefore P \wedge Q$
Disjunctive Addition	$P \therefore P \vee Q$

1. Use truth tables to verify each logical equivalence and each inference rule above.

2. Use truth tables to verify each equivalence below.

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|------------------------------------------------------------------------------------|----------------------------------------------------------------------------------|
| (a) $P \vee Q \equiv (\neg P) \rightarrow Q$ | (f) $(P \rightarrow Q) \vee (P \rightarrow R) \equiv P \rightarrow (Q \vee R)$ |
| (b) $P \wedge Q \equiv \neg(P \rightarrow (\neg Q))$ | (g) $(P \rightarrow R) \vee (Q \rightarrow R) \equiv (P \wedge Q) \rightarrow R$ |
| (c) $\neg(P \rightarrow Q) \equiv P \wedge (\neg Q)$ | (h) $P \leftrightarrow Q \equiv (P \wedge Q) \vee ((\neg P) \wedge (\neg Q))$ |
| (d) $(P \rightarrow Q) \wedge (P \rightarrow R) \equiv P \rightarrow (Q \wedge R)$ | (i) $P \leftrightarrow Q \equiv (\neg P) \leftrightarrow (\neg Q)$ |
| (e) $(P \rightarrow R) \wedge (Q \rightarrow R) \equiv (P \vee Q) \rightarrow R$ | (j) $\neg(P \leftrightarrow Q) \equiv P \leftrightarrow (\neg Q)$ |

3. Use a string of logical equivalences to verify each equivalence claimed in question 2.

4. Use disjunctive normal forms to verify each equivalence claimed in question 2.